I. Merhasin Y. Pinhasi M. Volshonok A. Gover 03.03.03

Append.C: QuadOpt: A MathCad program for Excitations Current Optimization using gaussian beams representation

Part II: Quad excitation calculation - solution #2

II.1 Description of the program

QuadOpt calculates the excitation currents I1, I2, I3, I4 of the four quadrupoles, which focus the beam from the acceleration tube exit into the wiggler.

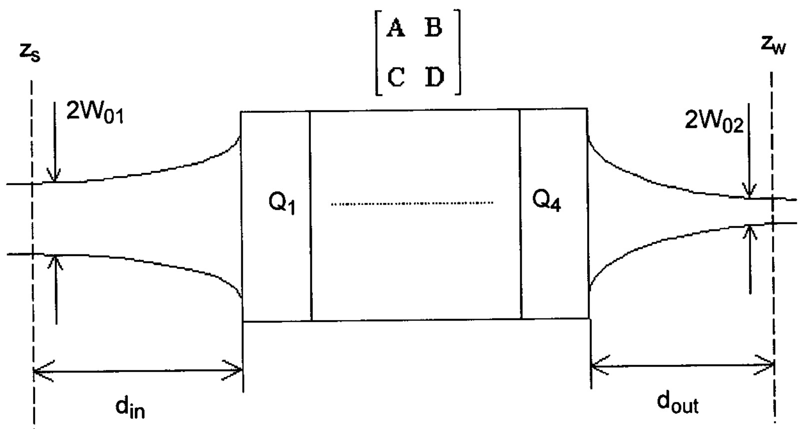


Fig. 1 Schematic of the beam-focusing model

The program is based on the Gaussian beam model in optics (using the analogy and the ABCD law.

The ABCD matrix of the quad system is defined between the "effective" (not geometrical) ends of the first and fourth quads. It is composed of analytical expressions for the paraxial ABCD matrices of the quads as function of their currents.

Assuming that we know the waist dimensions and positions before and after the quads: W_{01x}, W_{01y}, W_{02x}, W_{02y}, $d_{inx}=d_{iny}=d_{in}$, d_{outx}, d_{outy}. The program writes an equation relating the input and output waist dimensions using the ABCD law, and solves it for the excitation currents of the four quad currents. The solution is performed by an iterative process starting from an initial "guess" set of currents. From symmetry considerations two sets of solutions are expected.

Notes:

1. See p3 (item 6) for description of an ELOP procedure for calculating the goal output (virtual) waist parameters W_{02x}, W_{02y} and their positions Z_{wx}, Z_{wy}. (Z_{wx}, Z_{wy} are the waist positions relative to the center of the wiggler, from which the program calculates d_{outx}, d_{outy}).
2. In the present version it is assumed that $d_{inx}=d_{iny}=d_{in}$, and that the beam is at its waist exactly at the position of the screen S_1 located right after acceleration tube.
3. Because presently both the wiggler and the quads setting is symmetric relative to the wiggler center, the same program can be used to determine also the currents I_5-I_8: by symmetry $I_5=I_4$, $I_6=I_3$, $I_7=I_2$, $I_8=I_1$. Alternatively one can find a different solution, if different waist size W_{01} and position coordinate (Z_s) are required at the deceleration tube entrance, by operating QuadOpt again with the appropriate parameters.

II.2 Input Parameters

The parameters that frequently changed are listed here. All axial coordinates are relative to the wiggler center (absolute values).

Insert the desirable parameters (metric system):

$\varepsilon := \pi \cdot 2.223033346 \cdot 10^{-5}$ [m*rad] -emittance

$Zs := 2.681$ [m]

$w01x := 0.0075$ [m]

$w01y := 0.0075$ [m]

$Zwx := 0.600$ [m]

$Zwy := 0.544$ [m]

$w02x := 0.0010965$ [m]

$w02y := 0.00108$ [m]

Insert the initial "guess' currents set:

$I1 := 1.5$ [A]

$I2 := 1.2$ [A]

$I3 := 1.25$ [A]

$I4 := 0.7$ [A]

II.3 Fixed Input Parameters

The parameters that are not changed often are:

General Data:

$c := 2.9979 \cdot 10^{8}$	[m/s]	- velocity of light
$qe := 1.6022 \cdot 10^{-19}$	[coul]	- electron charge
$me := 9.1095 \cdot 10^{-31}$	[Kg]	- electron mass

$i := \sqrt{-1}$

Electron Beam Parameters:

$V := 1.4 \cdot 10^{6}$		[V]	- beam energy
$\gamma := \frac{qe \cdot V}{me \cdot c^{2}} + 1$	$\gamma = 3.73978156187106$		- relativistic γ
$\beta := \sqrt{1 - \frac{1}{\gamma^{2}}}$	$\beta = 0.964$		- relativistic b

II.4 Quads Configuration

$dB\_drdI := 0.15221$		[Tesla/(A*m)]	- quad magnetic field gradient
$a := \frac{qe \cdot dB\_drdI}{\gamma \cdot me \cdot \beta \cdot c}$	$a = 24.781$	$[1/(Am)^2]$	- quad general parameter

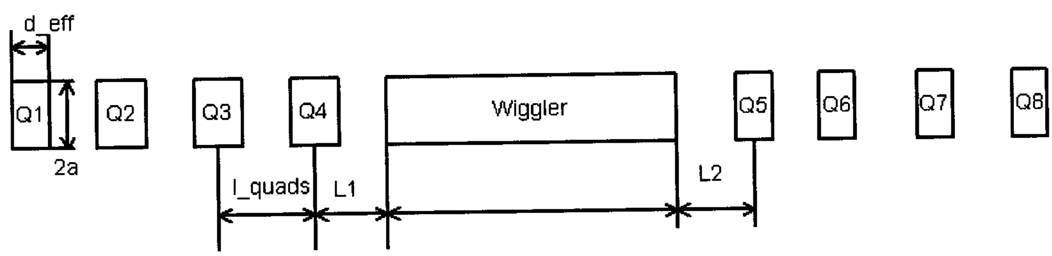


Fig.2 Quads configuration parameters

Quads Configuration Dimensions

$l\_quads := 0.345$ [m]

$L1 := 0.497$ [m]

$L2 := 0.497$ [m]

$Lwig := 1.200$ [m]

$d\_eff := 0.14$ [m] - quad width

$din := 2.681 - \frac{Lwig}{2} - L1 - 3 \cdot l\_quads - \frac{d\_eff}{2}$ $din = 0.479$ [m] - input free-space

$d := l\_quads - d\_eff$ $d = 0.205$ [m] - free_space between quads

del x,y is the position of the beam virtual waist in the x,y dimension relative to the end of the wiggler.

$delx := \frac{Lwig}{2} - Zwx$ $dely := \frac{Lwig}{2} - Zwy$

Output Distance:

$doutx := \left(L1 - \frac{d\_eff}{2}\right) + delx$ $doutx = 0.427$ [m]

- output free-space

$douty := \left(L1 - \frac{d\_eff}{2}\right) + dely$ $douty = 0.483$ [m]

II.5 Calculations of Quads System ABCD Matrix

Elements matrices:

$$Din := \begin{pmatrix} 0 & din \\ 1 & 1 \end{pmatrix} \quad Doutx := \begin{pmatrix} 0 & doutx \\ 1 & 1 \end{pmatrix} \quad Douty := \begin{pmatrix} 0 & douty \\ 1 & 1 \end{pmatrix} \quad D := \begin{pmatrix} 0 & d \\ 1 & 1 \end{pmatrix}$$ - free space matrices

$$Mc(I) := \begin{pmatrix} \cos\left(\sqrt{a \cdot I} \cdot d\_eff\right) & \frac{1}{\sqrt{a \cdot I}} \cdot \sin\left(\sqrt{a \cdot I} \cdot d\_eff\right) \\ -\sqrt{a \cdot I} \cdot \sin\left(\sqrt{a \cdot I} \cdot d\_eff\right) & \cos\left(\sqrt{a \cdot I} \cdot d\_eff\right) \end{pmatrix}$$ - quad converging matrix

$$Md(I) := \begin{pmatrix} \cosh\left(\sqrt{a \cdot I} \cdot d\_eff\right) & \frac{1}{\sqrt{a \cdot I}} \cdot \sinh\left(\sqrt{a \cdot I} \cdot d\_eff\right) \\ \sqrt{a \cdot I} \cdot \sinh\left(\sqrt{a \cdot I} \cdot d\_eff\right) & \cosh\left(\sqrt{a \cdot I} \cdot d\_eff\right) \end{pmatrix}$$ - quad diverging matrix

4 quads x- and y- ABCD matrices:

$$M4qx(I1,I2,I3,I4) := \begin{pmatrix} 1 & doutx \\ 0 & 1 \end{pmatrix} \cdot Mc(I4) \cdot \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot Md(I3) \cdot \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot Mc(I2) \cdot \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot Md(I1) \cdot \begin{pmatrix} 1 & din \\ 0 & 1 \end{pmatrix}$$

$$M4qy(I1,I2,I3,I4) := \begin{pmatrix} 1 & douty \\ 0 & 1 \end{pmatrix} \cdot Md(I4) \cdot \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot Mc(I3) \cdot \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot Md(I2) \cdot \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot Mc(I1) \cdot \begin{pmatrix} 1 & din \\ 0 & 1 \end{pmatrix}$$

$$q1x := i \cdot \pi \cdot \frac{w01x^2}{\varepsilon}$$

-Initial qx and qy

$$q1y := i \cdot \pi \cdot \frac{w01y^2}{\varepsilon}$$

$$q2x(I1,I2,I3,I4) := \frac{M4qx(I1,I2,I3,I4)_{0,0} \cdot q1x + M4qx(I1,I2,I3,I4)_{0,1}}{M4qx(I1,I2,I3,I4)_{1,0} \cdot q1x + M4qx(I1,I2,I3,I4)_{1,1}}$$

-final qx and qy

$$q2y(I1,I2,I3,I4) := \frac{M4qy(I1,I2,I3,I4)_{0,0} \cdot q1y + M4qy(I1,I2,I3,I4)_{0,1}}{M4qy(I1,I2,I3,I4)_{1,0} \cdot q1y + M4qy(I1,I2,I3,I4)_{1,1}}$$

$q\_2x(I1,I2,I3,I4) := \frac{1}{q2x(I1,I2,I3,I4)}$ $q\_2y(I1,I2,I3,I4) := \frac{1}{q2y(I1,I2,I3,I4)}$

$q\_2x(I1,I2,I3,I4) = 8.44 - 5.649i$

$q\_2y(I1,I2,I3,I4) = -1.448 - 0.651i$

-final parameters values

II.6 Solution#1 for I_1, I_2, I_3, I_4

Given

$$\frac{M4qx(I1,I2,I3,I4)_{0,0}}{M4qx(I1,I2,I3,I4)_{1,1}} = \left(\frac{w02x}{w01x}\right)^2$$

$$\frac{M4qy(I1,I2,I3,I4)_{0,0}}{M4qy(I1,I2,I3,I4)_{1,1}} = \left(\frac{w02y}{w01y}\right)^2$$

$$-\frac{M4qx(I1,I2,I3,I4)_{0,1}}{M4qx(I1,I2,I3,I4)_{1,0}} = \left(\frac{\pi}{\varepsilon}\right)^2 \cdot (w01x \cdot w02x)^2$$

$$-\frac{M4qy(I1,I2,I3,I4)_{0,1}}{M4qy(I1,I2,I3,I4)_{1,0}} = \left(\frac{\pi}{\varepsilon}\right)^2 \cdot (w01y \cdot w02y)^2$$

$I3 > 0$

$I := Find(I1,I2,I3,I4)$

$$I = \begin{pmatrix} 1.67594 \\ 1.22262 \\ 1.28847 \\ 0.65607 \end{pmatrix}$$

-Solution

II.7 Check Solution

A. This confirms that q(Zw)=iZR (waist position)

$q\_2x(I_0, I_1, I_2, I_3) = -18.49i$ $\quad q\_2x := \frac{1}{q\_2x(I_0, I_1, I_2, I_3)}$ $\quad ZRx := \pi \cdot \frac{w02x^2}{\varepsilon}$

$q\_2y(I_0, I_1, I_2, I_3) = -19.059i$ $\quad q\_2y := \frac{1}{q\_2y(I_0, I_1, I_2, I_3)}$ $\quad ZRy := \pi \cdot \frac{w02y^2}{\varepsilon}$

$q\_2x = 0.054i$ $\quad ZRx = 0.054$

$q\_2y = 0.052i$ $\quad ZRy = 0.052$

B. Current Variation

Here we vary I_4 only, keeping I_1-I_3 fixed, and confirm that the spot sizes $W_{2x}(Z_{wx})$ $W_{2y}(Z_{wy})$ are minimal for our I_4 solution.

$$q2x(I1, I2, I3, I4) := \frac{M4qx(I1, I2, I3, I4)_{0,0} \cdot q1x + M4qx(I1, I2, I3, I4)_{0,1}}{M4qx(I1, I2, I3, I4)_{1,0} \cdot q1x + M4qx(I1, I2, I3, I4)_{1,1}}$$

final qx and qy

$$q2y(I1, I2, I3, I4) := \frac{M4qy(I1, I2, I3, I4)_{0,0} \cdot q1y + M4qy(I1, I2, I3, I4)_{0,1}}{M4qy(I1, I2, I3, I4)_{1,0} \cdot q1y + M4qy(I1, I2, I3, I4)_{1,1}}$$

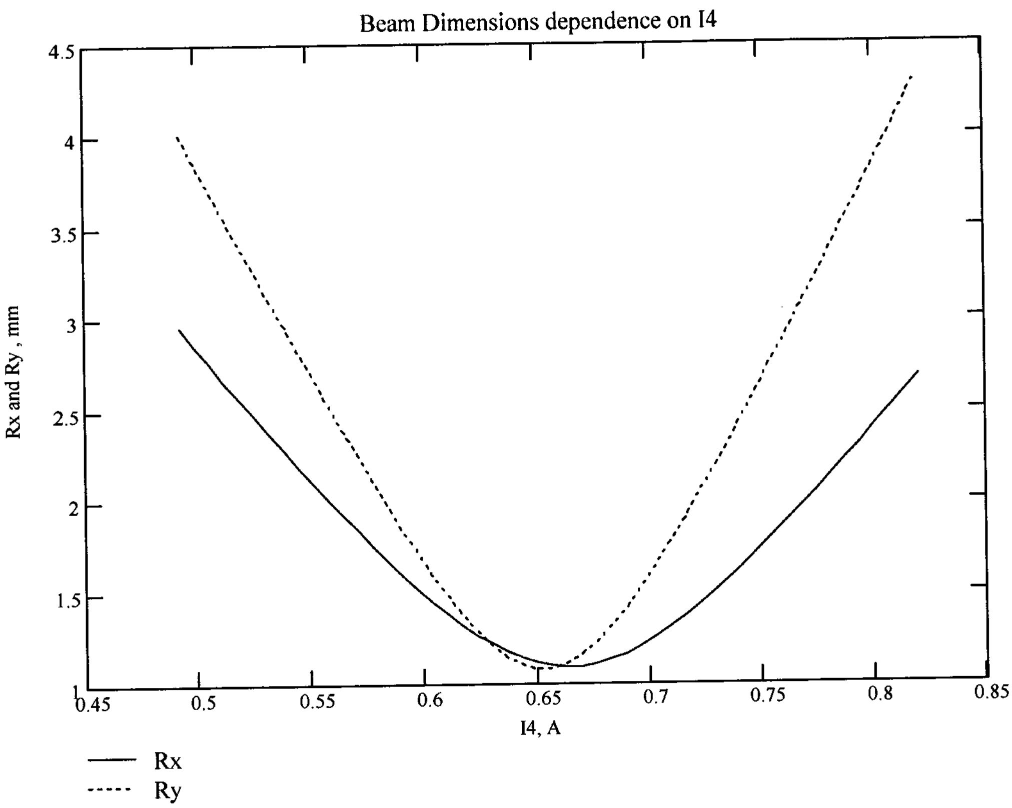
$q\_2x(I1, I2, I3, I4) := \frac{1}{q2x(I1, I2, I3, I4)}$ $\quad q\_2y(I1, I2, I3, I4) := \frac{1}{q2y(I1, I2, I3, I4)}$

$rx(I1, I2, I3, I4) := \sqrt{\frac{\varepsilon}{-\pi \cdot Im(q\_2x(I1, I2, I3, I4))}}$ $\quad ry(I1, I2, I3, I4) := \sqrt{\frac{\varepsilon}{-\pi \cdot Im(q\_2y(I1, I2, I3, I4))}}$

$I1 := I_0$ $\quad I2 := I_1$ $\quad I3 := I_2$

$k := 0..50$ $\quad I4_k := I_3 + (k - 25) \cdot \frac{I_3}{100}$

Beam Size as function of quads currents



I1 = 1.676

I2 = 1.223

I3 = 1.288

I4 = 0.656